**Presentation Notes**

**Why Study Turbulence**

* common phenonem
  + describe the motion of air over an airfoil
  + water around a ship
  + tornado
  + hurricane
* no analytics theory
  + Closure problem, The number of unknowns is larger than the number of equations, When we start deriving equations for unknowns, more variables appear
  + Total statistical description of turbulence requires an infi- nite set of equations
  + still not well understood
  + The [Navier–Stokes equations](https://en.wikipedia.org/wiki/Navier–Stokes_equations) govern the velocity and pressure of a fluid flow. In a turbulent flow, each of these quantities may be decomposed into a mean part and a fluctuating part, the reynolds stress.
  + nonlinearity of the Navier–Stokes equations means that the velocity fluctuations  still appear in the mean flow equations, trying to remove the nonlinearlity would yield an infinite hierarchy of equations for higher order nonlinear quantitie
* computational intractable
  + a true DNS of where th NS eqns are solved without any turbulence, This means that the whole range of spatial and temporal scales of the turbulence must be resolved in some data structure , from CDF-line a 3d DNS requires N^3 mesh of points where n^3 is greater than Re^9/4
  + memory requirements increase very fast for high reynolds number
  + integration of the solution must be done with a small time step such that a fluid particle moves only a very small distence in the mesh grid per iteration
* it has attracted the attention of some of the best minds in history

**Context**

* thnking of the tracer as something we can track and measure in the the flow and thinking of fluid flow as a collection of small particles particle we want to model the deformation/rotation of the particle as it moves and also how the particles separate as the flow evolves, this is the the growth rate
  + extract simulated data from an idealized flow
  + extract the PDF of angles and growth rate of the tracer wihtin the flow

**Advection/Diffusion**

* The ad**vection–diffusion equation** is a combination of the [diffusion](https://en.wikipedia.org/wiki/Diffusion_equation)and [advection](https://en.wikipedia.org/wiki/Advection_equation) equations,
* describes physical phenomena where particles, energy, or other physical quantities , here a scalar tracer move
* diffusion is the movement of tracer particles from high concentrations to low. concentrations
* advection is the motion of the particles alog the direction of the bulk flow

**Vorticity**

* The **vorticity equation** of [fluid dynamics](https://en.wikipedia.org/wiki/Fluid_dynamics) describes evolution of the [vorticity](https://en.wikipedia.org/wiki/Vorticity) **ω** of a particle of a [fluid](https://en.wikipedia.org/wiki/Fluid_dynamics) as it moves with its [flow](https://en.wikipedia.org/wiki/Flow_(fluid)), that is, the local rotation of the fluid
* derived by taking the curl of the NS equation
* F & D are focing and damping terms that are added to prevent a build-up of energy at large scales. The combination of damping and focring yeild a statistically steady state. Therefore we can regard the late time stats of the orientation angles as begin drawn from the equilibrium distribution

**Orientation Dynamics**

* to understand the alignment dynamics , we must compute the PDF of the angle X and the “growth rate”/”lyapunov exponent”/”stretching exponents”
* the model is based on the alignment dynamics of the tracer gradient with the straing (deforming) direction of the flow
* the model estimates the growth rates for a statistically homogenous, temporally varying flow, focus on the gradients of the concentation of the tracer (theta) in 2 dimensions
* this equation is derived from the advection diffusion equation by defiing B = (-theta, theta), working through in 2 dimenions and combining in matrix form
* grad theta = B^2, the magnitude can be calculated, we need to understand the angle of the direction.
* Writing (1) interms using the material derivate and taking dot product of both sides results in yields an expression for

**Stochastic Model**

* the var X represents the angle varphi
* Y is the fluctuations in the strain rate
* Z is the fluctuations in the orientation angle
* to understand the alignment dynamics , we must compute the PDF of the angle X and the “growth rate”/”lyapunov exponent”/”stretching exponents” these distributuons follow from the FP eqn for the PDF of the triple (X,Y,Z)
* the fluctuations are modeled as Ornstein/ulhenbeck processes
  + normally distributed
  + markov process (probability of the next event only depends on the previous ebent) The stationary distribution for an irreducible recurrent CTMC is the probability distribution to which the process converges for large values of *t*.
  + temporally homogeneous
* OU temporally homogeneous,
* the stationary PDF of an OU process statisifys the FP equation
* the corresponfing FP is .....
* where P is a function of X,Y,Z
* LOU is the OU operator
* FP equation gives you the probability distribution of the solutions to the SDE.
* the PDF of the alignment dynamics is found by computing the marginal PDF of X

**Numerical Sim**

* **Vorticity**
  + periodic boundary conditions are chosen as we dont care about the interaction with the boundaries, and want to simulate an inifinite system
  + discretizse in vorticity equation ,
  + transform to its fourier represnettion to levergae the numerical performance benefits of the FFT
  + save omaga at regular intervals to extract the PDF of the angle zeta
  + extract the PDF from the saved omega datasets once they are statistically homogenous , this is a visual inspection of the variance of omega plotted agaon
* **Fokker Plank**
  + solving for the PDF directly from the SDE results in very slow convergence , as we have to average over many simulations
  + solve the stationary distribution of the FP
  + extract the marginal distributions
  + X marginal is the PDF of the angle
  + growtj rate PDF is compted through a

**Analysis**

* **compare PDF from vorticity simulation to the PDF of the FP marginal of X**

**Conclusions**

* **aa**